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AUTHOR Flake, Janice L.
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ABSTRACT

This paper discusses six aspects of current practice in mathematics instruction of elementary school children: (1) use of symbolization, (2) use of models, (3) consideration of child development, (4) "logical" vs. "natural" approaches, (5) individualization, and (6) diagnosis and prescription. The author argues that symbols are frequently introduced prematurely, and that discovery approaches are more appropriate than logical ones for this level of development. (SD)

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WHAT MAKES MATHEMATICAL SENSE TO CHILDREN?

by

Dr. Janice L. Fiske
Mathematics Educ[illegible]
Florida State [illegible]

[illegible]
Washington
July [illegible]

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Dr. Janice L. Flake
Mathematics Education
Florida State University

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ABSTRACT

This paper takes an examination of present day practices concerning children's mathematics programs. Examination is given to: (1) premature use of symbolization, (2) the importance of models, (3) consideration of child development, (4) logical approach to mathematics versus a child's natural approach to mathematics, (5) individualization -- some problems, and (6) diagnosis and prescription.

It is felt that child development should be taken into consideration in diagnosis and prescription, and often it is not. It is also felt that instruction should more carefully follow discovery methods rather than a nice neat logical approach. Students need to bring organization to their own environment.

DO YOU HAVE MATHOPHOBIA?

DOES THE THOUGHT OF MATH SCARE YOU?

Many adults are "turned off" by mathematics. Often they say that they liked mathematics "until we did so-and-so in school"--they were exposed to some topic that seemed particularly difficult. Often the enjoyment of mathematics never returned after the hard topic was past.

Examination of Florida's state assessment test in 1974-1975 reveals that students have serious problems in the following content: (1) place value, (2) division, (3) fractional or rational numbers, (4) decimals, (5) perimeter, and (6) area. In addition, students over-all indicated weakness in problem solving. Most discouraging is the downward trend in performance levels from 3rd grade to 6th grade to 9th grade.

Examination of the 1972-73 National Assessment of Educational Progress reveal much the same patterns. Children seem to be lacking a basic "number sense." Severe concern was expressed that with the back-to-basics philosophy and accountability procedures even more emphasis will be placed on skills of number and symbol manipulation. Additional problems were identified in the area of consumer mathematics.

SO WHY ARE CHILDREN HAVING THESE PROBLEMS?

Through the FSU/FAMU - Leon County Teacher Corps project, careful examination of these issues have been given. Attention

to the elementary mathematics curriculum has been an outgrowth of the testing of a needs assessment model for in-service teacher education. A felt need identified by teachers from Riley Elementary School, Tallahassee, Florida, the project site school, was an improvement in their mathematics program. Possible problems were carefully examined, and alternative solutions were considered. As a result of this work, an alternative mathematics program has been born. The alternative program is now being considered on the county level, and shows the potential of growing much larger.

WHAT HAVE BEEN THE KEY ISSUES?

A number of key issues have been considered, many rejected and others emphasized. Factors to be discussed here are: (1) premature use of symbolization, (2) the importance of models, (3) consideration of child development, (4) logical approach to mathematics versus a child's natural approach to mathematics, (5) individualization--some problems, and (6) diagnosis and prescription.

Premature use of symbolization

As mentioned in the Florida State Assessment and in the National Assessment of Educational Progress, children are lacking a basic "number sense." Careful examination of a number of children through individual interviews reveals that often the children have learned rules for manipulating symbols, but that they do not really realize that the symbols relate to ideas that can be represented in a concrete

form. Symbols are simply a shorthand way of communicating about ideas . . . it is the ideas that should be taught and stressed.

On the one hand, some children eventually become hopelessly lost in the tangle of symbols and manipulation rules, as they accumulate through several years of curriculum. On the other hand, some children become quite facile with symbolic manipulation and are impatient with attempts to help them see the significance or application of these manipulations. In both cases, the children gain a distorted view: mathematics is seen as a formal stylized activity with symbolic stimuli and symbolic responses. The more mature view of mathematics as a powerful conceptual tool and as a way of making sense and analyzing real world phenomena is not developed. The logical aspect of mathematics, the use of mathematical technique for identifying and solving problems is not appreciated. A too early introduction of symbolism to children seems to result in this distorted view of what mathematics is.

When the ideas seem to be well developed, then shorthand ways of communicating about them can come in through the use of symbols. Careful attention is needed to make the correct "connecting" between the ideas and the symbols.

The importance of models

Models are very important in the teaching of mathematics. They help build the ideas about which the symbols are

communicating. Key ideas we have tried to develop in our new program are:

1. What are the abstract ideas that we want to teach?
2. What are models for those abstract ideas?
3. When have children conceived of those models?

Abstract ideas that are normally taught in elementary mathematics are: number, numeration (including place value concepts), rational numbers, operations on the numbers, and so forth. For instance, models for whole numbers may be of a counting type (5 objects to represent the number 5) or of a measuring type (a rod 5 times as long as a unit rod to represent the number 5). Models for whole numbers emphasizing their place value aspects may be literal or primitive (straws bundled into groups of ten) or conventionalized(money or the abacus). Models for fractional number may be of a measurement type (1/4 of a rectangle shaded to represent 1/4), or a counting type (1 of 4 objects shaded to represent 1/4), or of a ratio type (a set of objects compared to a set of r to represent 1/4).

When have children conceived of these models? It turns out that it is not always obvious that a child is able to hold fiveness in his head. Give a child five objects, ask him/her to count the objects. Move the objects around, clearly not adding any nor taking away any objects. Then ask him/her how many objects there are. Some children are not able to hold this fiveness in their heads. At which point does it really make sense to force the child to learn

basic addition and subtraction facts when he/she can not even hold onto fiveness in his/her head?

Similarly, it is not always clear to a child that 1 ten and ten ones are the same. Yet we often ask them to "carry" and borrow." Does that make sense?

Likewise, we have found that a child can have fourths of equivalent pieces, and yet does not see them as being the same. Does it make sense to be concerned with adding and subtracting fractions at this point? Does it even make sense to be concerned with comparing fractions or with equivalent fractions?

Consideration of child development

Fifty years of intensive study of children and their conceptions of various mathematical models has been conducted by Jean Piaget and his co-workers. They have found that the children all go through certain stages of development. They go through the same stages. Children in the age of 5 to 10 are going through stages that move from operating at a highly perceptual level to a conceptual level. In the early stage the child will react to what he/she sees "out there" without having an internalized model of what he/she sees. For instance, take two sets of objects, could be checkers, candy, or whatever, in this case we will use "X" and "O." First match them together in a one-to-one correspondence. Ask the child if there are the same number of each type. If he/she says "yes," then spread one of the rows apart (See Figure 1).

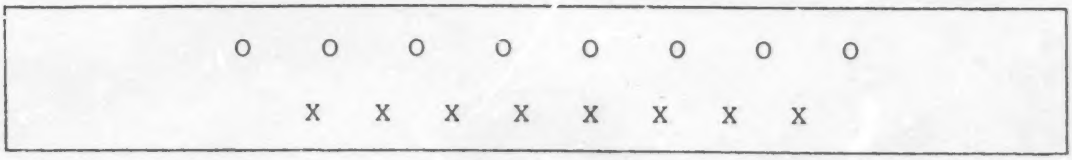

Figure 1

Again ask the child if each type has the same amount. If the child says "no," then he/she is said to be a non-conserver of number, where "conservation" means the ability to see that the attribute of number has remained the same.

There are similar tests of conservation of length, volume, area, quantity, and so on.

Is conservation, or the ability to hold these ideas in one's head, related to success in mathematics? A number of research studies have been conducted to examine this. The relationship of the ability to conserve on Piagetian tasks to achievement in mathematics was conducted by Rohr (1973). She found that conservers of area scored significantly higher than non-conservers on concepts, problem solving, and total mathematics. Conservers of volume scored significantly higher than non-conservers on all of the mathematics achievements that were measured. Overall, she found that conservation skills do relate to achievement in mathematics, and more advanced conservation skills appeared to relate more directly than the elementary ones.

Bearison (1975) found that early conservation of liquid quantity served as a good predictor for later mathematics achievement. Wheatley (1967) found conservation of number being more highly related to achievement that counting or

cardination; he also found that counting by ones did not seem to be highly related to achievement for first-grade students.

Studies done by Steffe (1966) and by VanEngen and Steffe (1966) indicate that first graders can learn to work correctly symbolic sentences such as, 2 + 3 =, 4 + 5 =, without being conservers of number. But it appeared that they were working with memorized facts rather than relating to models in their heads. Many of the children were unable to abstract the number concept from physical situations. Concern was expressed that caution be used about pressing the children to know facts before they are ready.

One can get by with low level accomplishments by memorizing one's way through them. But when one must analyze, synthesize, evaluate, and apply those low level skills, memorization will not carry one through. Let's reexamine the 1974-1975 Florida State-Wide Assessment and the 1972-73 National Assessment of Educational Process results. Perhaps one of the serious problems that children are being forced into is premature symbolization and pressing for facts, when models have not been properly developed in their heads. They can succeed in lower level performances, but having a weak foundation, as the building becomes more and more complex it may well crumble.

Considerable research has been conducted studying "training" sessions for conservation. Results have shown that it is reasonably ineffective. It would appear that what is involved

is far more complex than simple stimulus-response training conditions.

Bruner (1971) commented,

> I am inclined to think of mental development as involving the construction of a model of the world in the child's head, an internalized set of structures for representing the world around him. (p. 103)

Flavell (1963) stressed:

> It just seems natural to assume that we see space as it is and have always seen it that way. What Piaget wants to stress is (1) that this effortless seeing is really the end product of long and arduous developmental construction, and (2) that construction itself is more dependent upon actions that upon perceptions per se. (p. 328)

Research[1] has shown:

(a) Children learn mathematical concepts more slowly than we realized. They learn by their own activities.

(b) Although children think and reason in different ways, they all pass through certain stages depending on their chronological and mental ages and their experience.

(c) We can accelerate their learning by providing suitable experiences, particularly if we introduce the appropriate language simultaneously.

(d) Practice is necessary to fix a concept once it has been understood, therefore practice should follow, and not precede, discovery, understanding, or internalization of concepts.

Key factors contributing to reaching the developmental levels involving the various conservation levels are: (1) maturation, (2) experience, (3) social transmission, and (4) self-regulation. As educators, we can do nothing

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[1] Schools Council Curriculum Bulletin No. 1, Mathematics in Primary Schools, 4th ed. London: Her Majesty's Stationery Office, 1972.

about maturation, or physiological growth, except recognize that it exists and to be patient. For the other factors, certainly we can give the child a number of experiences, discuss the ideas with him/her, and allow him/her to work through many of the ideas. It is a challenge to us, the educators, to become sensitive to when ideas are "making sense" to the child and where to go from there.

The first order of priority of the program we are developing is to build models in the children's minds. This is done through allowing him/her to "act on" objects, discuss ideas related to the objects, and build mathematical ideas. A child learns best through his own "actions" and needs many, many experiences with objects, blocks, clay, sand, etc. where he/she is classifying them, seriating them, building with them, and so on. These sorts of activities help build the kinds of models internally that can later be used to develop the abstract mathematical ideas.

As an example, fractional ideas are usually developed through the use of rectangular and circular models, which are really using area-measure. Yet little attention is given to the development of area prior to the use of those models. We plan to first carefully build the area concepts that can later be used to develop the fractional ideas.

Logical approach versus natural approach to mathematics

It was felt in the early 1960's that elementary mathematics should be taught in a logical and articulated sequence that

successively defines number operations in different ways for different sets of numbers, but in ways that show what properties are consistent from set to set and how a general, integrated, abstract concept of number grows out of these structural relationships such as commutative, associative, and distributive properties.

Since then a number of educators have challenged this underlying assumption. Max Beberman (1971), noted "Father of the New Math," commented,

> "I now think that it is a fallacy of mathematics curriculum development for young children that logical organization of the subject determines its pedagogical organization. When a child learns mathematics via firsthand experiences with real things, the reality of the context provides him with all he may need at that time to make sense out of what he is learning. A six-year-old trying to build a tower 10 inches high might discover than he can do this with a 6 1/2-inch block and a 3 1/2-inch block. He does not have to be taught how to add "mixed numbers" before he can make that discovery, nor would this be the time to teach him an algorithm for doing so." (p. 28)

Morris Kline, author of Why Johnny Can't Add (1973), commented,

> In many areas the present emphasis on the logical approach is sheer hypocrisy. What mathematician used the logical development of the complex number system to justify his operations with real or complex numbers? Yet this is what is taught to students as the way to learn the "truth" about numbers. (p. 56)

He also added,

> Every curriculum claims to teach the student how to discover results for himself. Discovery, contrasted with and opposed to the passive and uncritical acceptance of finished or polished statements of theorems and proofs, amounts to the

> creation or recreation of mathematics by the student, possibly with the guidance by the teacher. (p. 58)

A study conducted by Merrill et. al. (1969) found that in an experiment conducted with 160 children in the second, third, and fourth grades, although the hierarchy of concepts was logical, it did not represent the skills in a sequence acquired by these children.

There is a saying, "Ontogeny recapitulates phylogeny." Translated means, the life history of development of an individual organism, e.g., the child, repeats the development of the history of man. With that in mind, perhaps one major fallacy of a logical approach to an elementary mathematics program, is that too much has been externally organized for the child, and thus the organization is not his/her own internal organization. Perhaps, he/she has to go through the struggling of organizing ideas for himself/herself in order for the ideas to become internalized. It is the internalized organization that is important!

Individualization - some problems

Individualized instruction has materialized as a very ambiguous term. Ideally, given an individual with all of his/her own unique learning characteristics and/or learning styles, instruction is uniquely designed for the individual. Each child would have his/her own unique learning experiences.

What has happened, in fact, though, has been that materials have been prepackaged with a number of preset,

prepackaged diagnostic-prescriptive types of materials, in which the only thing that is really individualized is the rate one progresses through the materials.

One such set of materials is the IPI (Individually Prescribed Instruction) Mathematics program. This program is built on a sequence of 359 instructional objectives with a student booklet for each behaviorally specified skill. Research has failed to show that this program as well as many others of a similar nature produce any better results than whole class instruction.

In addition to other criticisms of the prepackaged individualized programs, critics maintain that (1) many of the problem-solving skills and attitudes considered to be part of a good mathematics program are missing; (2) most individualized programs deny the value of students discussing mathematical problems with each other and with a teacher; (3) the emphasis on testing until mastery tends to lead students to shallow learning of "local" rules and to emphasis on low-level skills; (Major concepts are difficult to generalize solely from paper and pencil activity, particularly at the lower grade levels.); and (4) a major difficulty with most diagnostic-prescriptive individualized programs is the quantity of required record-keeping.

How many of these materials are concerned with developing models in children's heads, then using the models to develop abstract ideas? Can the development of internalized models be done through carefully sequenced minute behavioral objectives?

Diagnosis and Prescription

In the state of Florida, a law has been passed that every child in K - 3 shall be diagnosed and given appropriate prescriptions. So how does one go about diagnosing? One possibility is to use a packaged program such as the IPI-Math program described earlier. But data has already told us that we should be cautious. There are a number of other similar programs now available on the market. . .what does data say about them? Can one carefully analyze skills in a minute fashion and tightly sequence the child through them?

Beberman (1971) commented,

> There are infinitely many things children can do in school. Some things will help him learn to learn, some will encourage him to think; some will help him find ways of expressing himself; and many will do nothing but waste his time, or worse, make him bitter about school. Mathematics as a school subject has the capacity for doing all these things, the negative ones as well as the positive ones. It is relatively easy to arrange things so that mathematics will promote only the negative outcomes. For example, you can design a curriculum in which the mathematical concepts are arranged in logical sequence and the algorithms are rationalized in terms of mathematical laws; then you parcel this out to children in small pieces, each to be mastered in its turn, but with provision for considerable remedial work to rescue the numerous children who have failed to learn. With such a program you can be sure that a child will have done plenty of memorizing (he will even have memorized his 'understanding'), that he will not have learned how to learn (the authors of the instructional materials will have arranged the 'optimal' instructional sequence), and that he will not have done much thinking. He will certainly have developed feelings about mathematics (and school) but will probably have not learned how to learn how to express them except under his breath.
>
> (pp. 27-28)

Indeed, in theory, it sounds good to carefully diagnose a child and to give him/her appropriate instruction. But caution should be used to diagnose the "right" behaviors. To immediately focus upon skills and expect a child to become proficient in skills when he/she does not even have the underlying concepts, or models in his/her head will only lead to frustration for the child.

Brownell (1944) commented,

> The four instructional weaknesses to which connectionistic theory has contributed directly or indirectly are:
>
> 1. Our attention as teachers is directed away from the processes by which children learn, while we are over-concerned about the product of learning.
>
> 2. Our pace of instruction is too rapid while we fail to give learners the aids they need to forestall or surmount difficulty.
>
> 3. We provide the wrong kinds of practice to promote sound learning.
>
> 4. Our evaluation of error and our treatment of error are superficial.
>
> (p. 148)

Distinction needs to be clearly made between concepts and skills. Much of present day curricula focuses primarily upon skills, as do most diagnostic-prescriptive programs. A typical first grade class will have approximately one-half of its students who are non-conservers of number. Yet, the usual curriculum has the children primarily focusing upon "paper and pencil" addition and subtraction of one-digit, and even two-digit numbers. Can all of that really make any sense to the children? This past year I spent a lot of

time in a third grade class, in which a number of the children still did not know their facts, nor could they make the most simple classifications or patterns. What has been happening for the past two years? Another first grade teacher, with whom I have been working, spent much of the first part of the year working with classification, seriation, patterns, and measuring with Cuisenaire rods, before moving into the addition and subtraction. This teacher's children started the year with the usual approximately half of her 24 students as non-conservers of numbers; by the end of the year, only four, or 1/6 of the children were non-conservers, and they did as well in arithmetic as other children who had spent all year on arithmetic. Hopefully, these children will have a foundation to build mathematics down the road.

A study by Brownell and Chazol (1935) showed that while intensive work was done with drill and practice with third graders, their intellectual levels did not significantly change. Caution should be used in pressing for skills too soon! At the same time sensitivity should be developed to know when to drill on skills.

Brownell (1947) commented:

> Meaningful arithmetic:
>
> 1. Gives assurance of retention.
> 2. Equips him with the means to rehabilitate quickly skills that are temporarily weak.
> 3. Increase the likelihood that arithmetical ideas and skills will be used.
> 4. Contributes to ease of learning by providing a sound foundation and transferable understandings.

5. Reduces the amount of repetitive practice necessary to complete learning.

6. Safeguards him from answers that are mathematically absurd.

7. Encourages learning by problem-solving in place of unintelligent memorization and practice.

8. Provides him with a versatility of attack which enables him to substitute equally effective procedures for procedures normally used but not available at the time.

9. Makes him relatively independent so that he faces new quantitative situations with confidence.

10. Presents the subject in a way which makes it worthy of respect. (pp. 100-101)

COMPONENTS OF THE PROPOSED PROGRAM

One of the major decisions we have made on our diagnostic-prescriptive curriculum is that major emphasis must be given to developmental levels. In addition to other studies quoted throughout this paper, recommendations for emphasis upon developmental diagnoses has been given by Smith (1973) and Irmen (1974). In addition, various levels of understanding such as identified by Brownell (1928) are to be included. Emphasis is to be upon preventive rather than corrective diagnosis.

Secondly, we are prescribing appropriate uses of manipulative aids and activities. Williams and Shuard (1970) commented:

> We can see that a child's awareness of mathematical properties comes from three types of experiences:

(a) Repetition of experiences of the same object or event;

(b) Contrast between two different things or events;

(c) His own manipulation of things or his observation of their behavior. (p. 8)

A careful analysis of appropriateness and uses of aids are currently being studied. There are many different levels of abstractions within the aids, as well as many different ways in which they can be used.

A third important part of our proposed program is that it must be related to the physical world. Considerable evidence has shown that a narrow focus upon number and symbolization alone, to the exclusion of the many aspects of mathematical ideas, will not in itself cause the children to make the transfer to real world applications. Furthermore, it is the real world experiences that help build the abstract ideas. Thus, the program should evolve from experiences with: (a) relationships (classification, seriation, patterns), (b) number, (c) measurement, (d) spatial construction and shapes, and (e) consumer mathematics.

A fourth important part of our proposed program must be focused at problem solving and discovery learning. Perhaps one of our major goals in school should be to help children to learn how to think and learn for themselves. It we do all the decision making, and organize everything for the children, when do they ever learn how to learn for themselves?

A fifth major component of our proposed program is in teacher education. In no way do we plan to prepare "teacher proof" materials that side step the teacher. We feel that it is extremely important that the teachers understand the underlying theories, rationale, and overall framework of a program that is to work effectively, and help build the program!

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